SCS 139

II.4 Alternating Current

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$$v_R = i_R R$$

$$i_C = C\frac{dv_C}{dt}$$

$$v_L = L\frac{di_L}{dt}$$

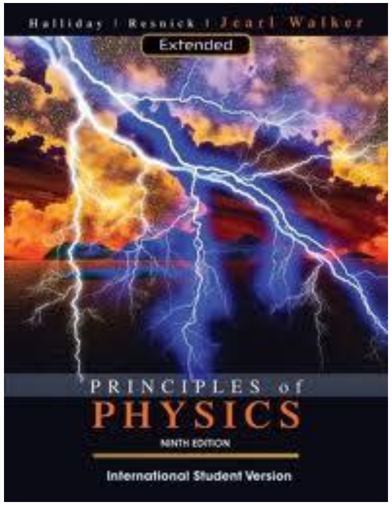
How will R, L, C behave in circuits under forced oscillation.

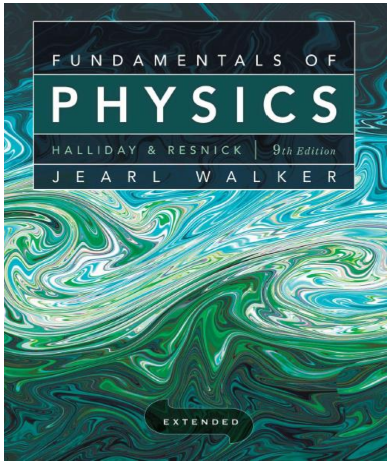
Office Hours:

Library (Rangsit)	Mon	16:20-16:50
BKD 3601-7	Wed	9:20-11:20

Reference

- Principles of Physics
- Ninth Edition, International Student Version
- David Halliday, Robert Resnick, and Jearl Walker

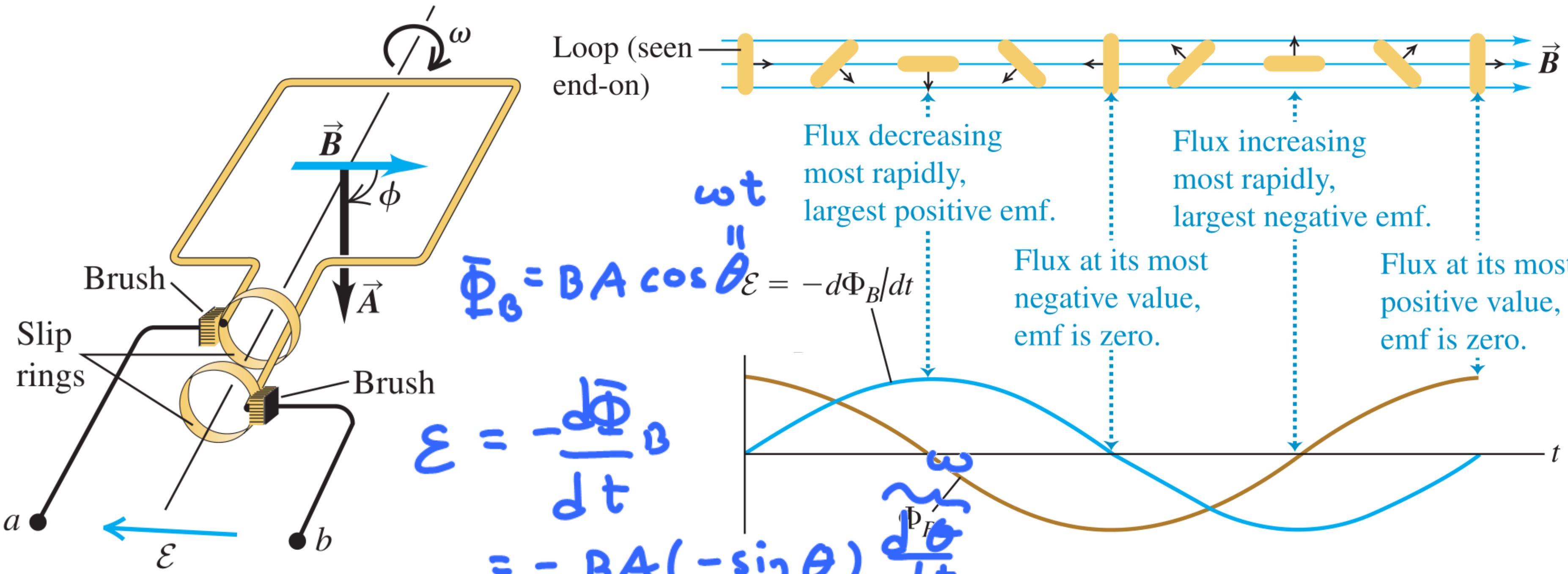




- Chapter 31
  - 31-6 Alternating Current
  - 31-7 Forced Oscillations
  - 31-8 Three Simple Circuits

Alternating-Current Generator

- A conducting loop rotates (with constant angular speed ω) in an external (uniform and constant) magnetic field.

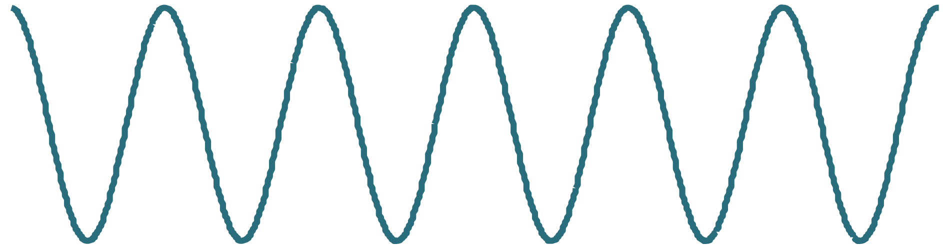


$$\Phi_B = BA\cos\theta, \quad \theta = \omega t$$

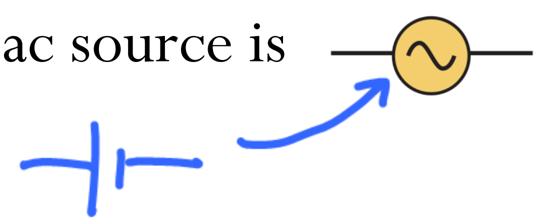
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -BA(-\sin\theta)\frac{d\theta}{dt}, \quad \frac{d\theta}{dt} = \omega$$

- Connections from each end of the loop to the external circuit are made by means of that end's slip ring.

Sinusoids



- A **sinusoid** (or sinusoidal signal) is a signal (e.g. voltage or current) that has the form of the sine or cosine function.
  - Turn out that you can express them all under the same notation using only cosine (or only sine) function.
    - **We will use cosine**.
- A sinusoidal current is referred to as **alternating current** (ac).
- Circuits driven by sinusoidal (current or voltage) sources are called **ac circuits**.
  - We use the term **ac source** for any device that supplies a sinusoidally varying voltage (potential difference) or current
- The usual circuit-diagram symbol for an ac source is



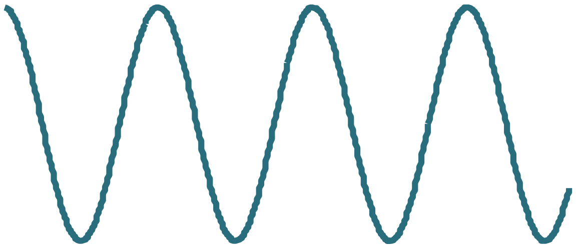
Sinusoids: Standard Form

- General sinusoidal signal (in cosine form)

$$x(t) = X_m \cos(\omega t + \phi) = X_m \cos(2\pi f t + \phi).$$

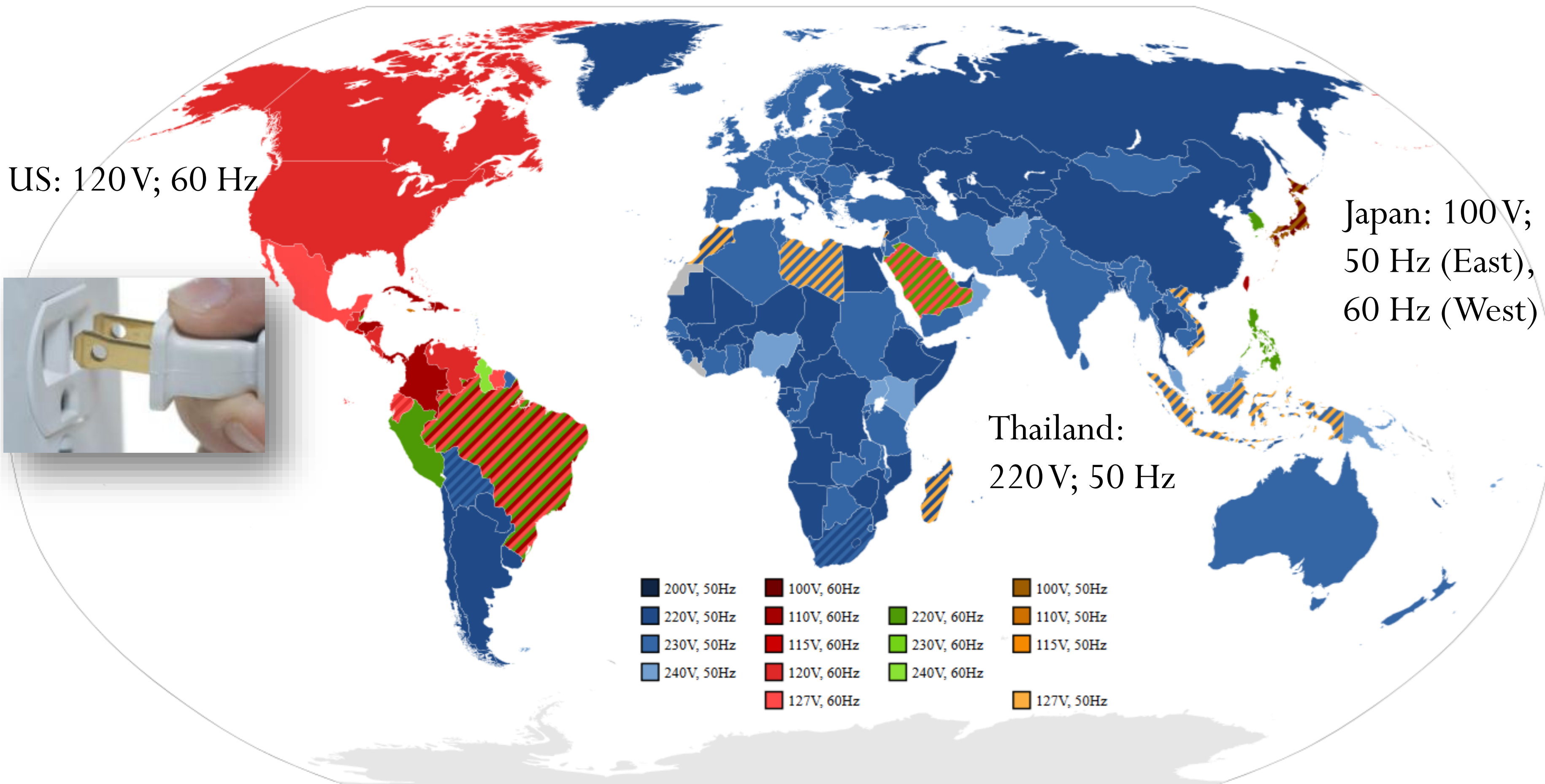
- X_m: amplitude of the sinusoid
  - Nonnegative when expressed in standard form
- T: period (the time of one complete cycle)
- f: frequency
  - #cycles per second or hertz (Hz)

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$



- ω: angular frequency in radians/s (or rad/s)
- ϕ: phase
  - Between -180° and $+180^\circ$ in standard form

Around the World: Voltages and Frequencies



US: 120 V; 60 Hz

Japan: 100 V;
50 Hz (East),
60 Hz (West)

Thailand:
220 V; 50 Hz

Conversions to standard form

- When the signal is given in the sine form, it can be converted into its cosine form via the identity

$$\sin(x) = \cos(x - 90^\circ).$$

  In particular,

$$X_m \sin(\omega t + \phi) = X_m \cos(\omega t + \phi - 90^\circ).$$

- We can avoid having X_m with negative sign by the following conversion:

$$-\cos(x) = \cos(x \pm 180^\circ).$$

  In particular,

$$-A\cos(\omega t + \phi) = A\cos(2\pi ft + \phi \pm 180^\circ).$$

  - Note that usually you do not have the choice between $+180^\circ$ or -180°. The one that you need to use is the one that makes $\phi \pm 180^\circ$ falls somewhere between -180° and $+180^\circ$.

Exercise

Express the following sinusoids in their standard forms

$$5\cos\left(2t-45^\circ\right) = 5\cos\left(2t+(-45^\circ)\right)$$

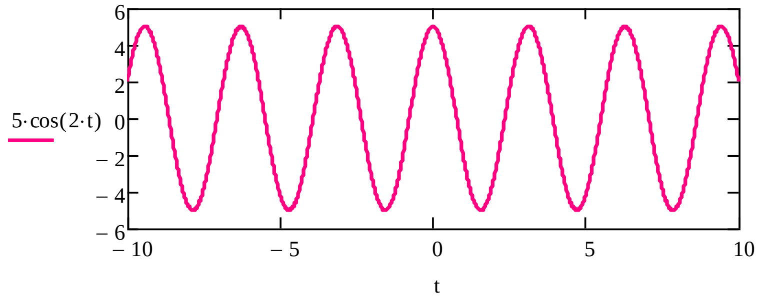
$$5\sin\left(\underbrace{2t-45^\circ}_{x}\right) = 5\cos\left(\underbrace{2t-45^\circ}_{x}-90^\circ\right) = 5\cos\left(2t+(-135^\circ)\right)$$

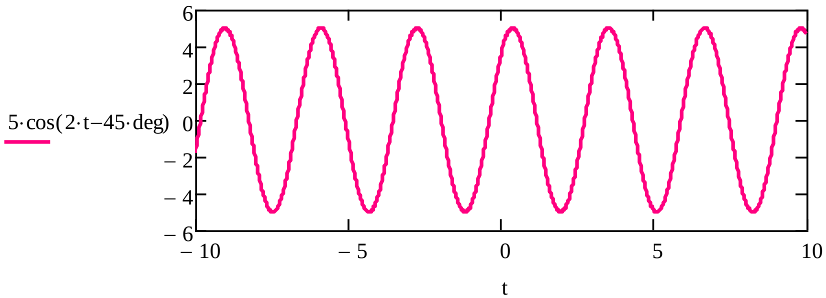
$$-5\cos\left(2t-45^\circ\right) = 5\cos\left(2t-45^\circ+180^\circ\right) = 5\cos\left(2t+135^\circ\right)$$

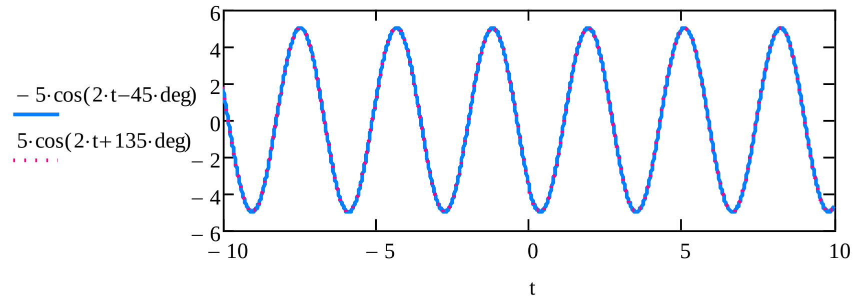
$$-5\sin\left(2t-45^\circ\right) = -5\cos\left(2t-45^\circ-90^\circ\right) = -5\cos\left(2t-135^\circ\right)$$

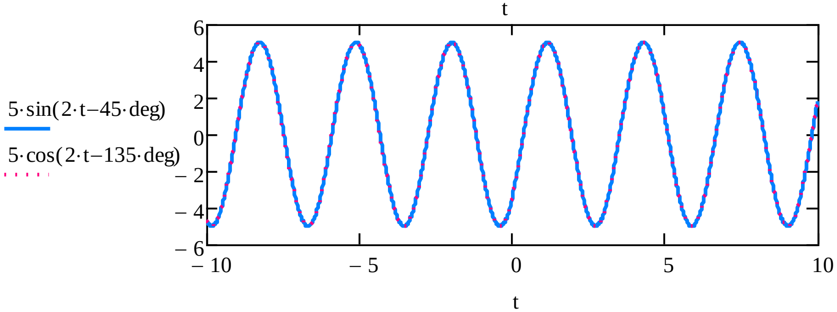
$$= 5\cos\left(2t-135^\circ+180^\circ\right) = 5\cos\left(2t+45^\circ\right)$$

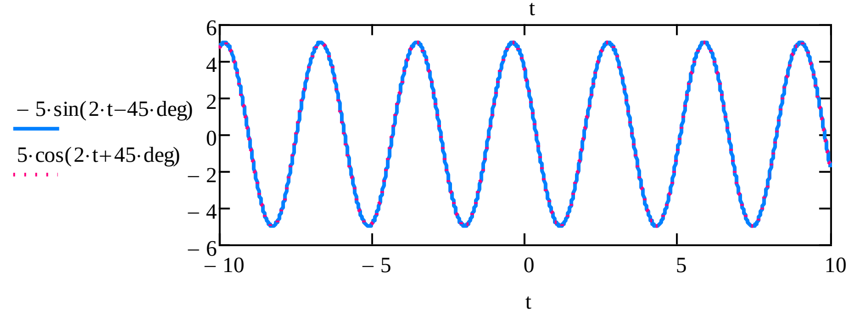
Exercise


5·cos(2·t)
t

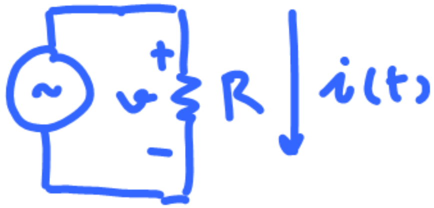

5·cos(2·t−45·deg)
t


− 5·cos(2·t−45·deg)
5·cos(2·t+135·deg)
t


5·sin(2·t−45·deg)
5·cos(2·t−135·deg)
t


− 5·sin(2·t−45·deg)
5·cos(2·t+45·deg)
t

$v(t) = V_m \cos(\omega t + \phi_v)$

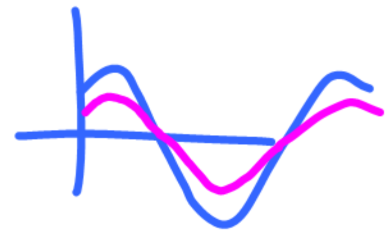


$v = iR$, where $v = V_m \cos(\omega t + \phi_v)$

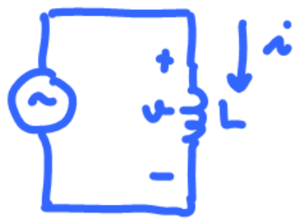
$i = \frac{v}{R} = \frac{V_m}{R}\cos(\omega t + \phi_v)$, where $i = I_m \cos(\omega t + \phi_i)$

Resistor: $V_m = I_m R$

$\phi_v = \phi_i$



$I_m \cos(\omega t + \phi_i)$



$V_m \cos(\omega t + \phi_v)$

$v = L\frac{di}{dt} = L\frac{d}{dt}\left(I_m \cos(\omega t + \phi_i)\right)$

$= L I_m \left(-\sin(\omega t + \phi_i)\right)\omega$

$= \omega L I_m \left(\cos(\omega t + \phi_i - 90° + 180°)\right)$

$= (\omega L) I_m \cos(\omega t + \phi_i + 90°)$

Inductor: $V_m = (\omega L) I_m$

$\phi_v = \phi_i + 90°$

Application: Measuring Body Fat by Bioelectric Impedance Analysis

- The electrodes attached to this overweight patient's chest are applying a small ac voltage of frequency 50 kHz.
- The attached instrumentation measures the amplitude and phase angle of the resulting current through the patient's body.
- These depend on the relative amounts of water and fat along the path followed by the current, and so provide a sensitive measure of body composition.